Math 53: Multivariable Calculus **Sections 212, 213**

Worksheet for 2020-09-21

Problem 1. Here are some conceptual questions on the gradient and directional derivatives.

(a) Is it possible for different level sets of a function to intersect?

(b) How are the direction and magnitude of the gradient vector related to level sets?

(c) If $\mathbf{r}(t)$ is a curve contained in the surface $f(x, y, z) = 0$, how are the vectors $\mathbf{r}'(3)$ and $\nabla f(\mathbf{r}(3))$ related? (Are they parallel? Orthogonal? Something else?)

(d) Fix a function $f(x, y)$, a number c, and a point (a, b) where $\nabla f(a, b) \neq \mathbf{0}$. How many unit vectors $\mathbf{u}$ are such that $D_{\mathbf{u}}(a, b) = c$? Hint: the answer depends on $|c|$.

Problem 2.

(a) Let $f(x, y)$ be a function on $\mathbb{R}^2$ and $\mathbf{r}(t)$ be an arc-length parametrized path in $\mathbb{R}^2$ (in other words, $|\mathbf{r}'(t)| = 1$ for all t).

Use the chain rule to show that

$$D_{\mathbf{r}'(t)} f(\mathbf{r}(t)) = \frac{\mathrm{d}}{\mathrm{d}t} f(\mathbf{r}(t)).$$

(b) Use the path $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$ to compute $f_y(1, 0)$ for $f(x, y) = \cos^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right)$.